Department of Mathematics

Course Profile

Course Number: MATH 323	**Course Title: Calculus of Variations**
Required / Elective: Elective	**Prerequisites:** None
Catalog Description: Historical approach to basic problems; variation of a functional; weak and strong extrema; Euler-Lagrange equations; variational derivative, higher order derivatives, subsidiary conditions; variable end point problems; broken extremals. Noether's heorem, Hamilton- Jacobi Equation, Jacobi's theorem; quadratic functionals, second variation of a functional. Direct methods, Ritz and Kantorovich methods.	**Textbook / Required Material:** Textbook: I.M.GELFAND & S.V. FOMIN, *Calculus of Variation,* Prentice Hall, 1963.

Course Structure / Schedule: (3+0+0) 3/ 7 ECTS

Elements of the theory: Functionals. Function spaces. Variation of a functional. Several variables. Euler Equation. Variable end point problems. Variational derivative. Invariance of Euler's Equation. Examples. *Further Generalizations*: Fixed end point problems. Parametric form. Higher order derivatives. Subsidiary conditions. *The General Variation of a Functional*: Basic Formula. Moving end points. Broken Extremals. Weierstrass-Erdmann Conditions. Examples. *The Canonical Form of Euler Equations and Related Topics*: Canonical form of EE. First Integrals of EE. Legendre Transformations. Examples. Canonical Transformations. Noether's Theorem. Principle of least action. Conservation Laws. Examples. Hamilton-Jacobi Equation. Jacobi's Theorem. Examples. *The Second Variation. Sufficient Conditions for Weak Extremum*: Quadratic functionals, second variation of a functional. Legendre's condition. *Fields. Sufficient Conditions for Strong Extremum*: Definitions. Field of functionals. Hilbert's invariant integrals. Strong extremum. Examples. *Direct Methods in the Calculus of Variations*: Minimizing sequences. Method of finite difference. Ritz method. Examples. The Sturm-Liouville problems. General review, more examples. *Variational Problems involving Multiple Integrals*: Variation of functionals on a fixed region. Continuous mechanical systems. Variation of a functional on a variable region. Applications to field theory. *Direct Methods in the Calculus of Variations*: Minimizing Sequences. Method of finite difference. Ritz method. Examples. The Sturm-Liouville Problems. General review, more examples.

Design content: None	**Computer usage:** Partly

Course Outcomes: By the end of the course the students should be able to:

1. give a modern treatment of the calculus of variations from a rigorous perspective, blending classical and modern approaches and applications. **[2,3, 6],**
2. learn rigorous results in the classical and modern calculus of variations and see possible behaviour and application of these results in examples. **[3, 6].**

[2] demonstrate knowledge of mathematics and mechanics to construct, analyze and interpret real world problems,
[3] demonstrate the ability to apply mathematics to the solutions of problems,

[6] have a basic knowledge of the main fields of mathematics and mechanics, including differential equations, elasticity theory, fluid mechanics,

Recommended reading:

1. U. Brechtken-Manderscheid, *Introduction to the Calculus of Variations* (Chapman & Hall, 1991).
2. H. Sagan, *Introduction to the Calculus of Variations* (Dover, 1992).
3. J. Troutman, *Variational Calculus and Optimal Control* (Springer–Verlag, 1995).

Teaching methods: Three hours theoretical presentation with illustrative problem solving.

Assessment methods:

Homework, quiz, midterm and final exams.

Student workload:

Pre-reading	35 hrs
Lectures ……	45 hrs
Preparatory reading ……..	35 hrs
Literature review for presentation……	45 hrs
Team work for presentation ……	15 hrs
**TOTAL …… **	**175 hrs …… to match 25x7 ECTS**

Prepared by: Prof.Dr.Esin İnan	Revision Date: 08.02.2010